

# Turunan fungsi implisit

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$y = f(x) \rightarrow$  fungsi eksplisit

1.  $x^3 - 3x^2y + y^2 = 0$

2.  $y + \sin(xy) = 1$

3.  $\tan(xy) - 2y = 0$

4.  $x^2 \sin(xy) + y = x$

$f(x, y) = 0 \rightarrow$  f. Implisit

$x^3 - 3x^2y + y^2 = 0$

$3x^2 dx - 6xy dx - 3x^2 dy + 2y dy = 0$

$3x^2 - 6xy \frac{dy}{dx} - 3x^2 \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$

$(-3x^2 + 2y) \frac{dy}{dx} = -3x^2 + 6xy$

$\frac{dy}{dx} = \frac{-3x^2 + 6xy}{-3x^2 + 2y}$

$y + \sin(xy) = 1$

$dy + \cos(xy) \cdot y dx +$

$\cos(xy) \cdot x dy = 0$

$\frac{dy}{dx} + y \cos(xy) + x \cos(xy) \frac{dy}{dx} = 0$

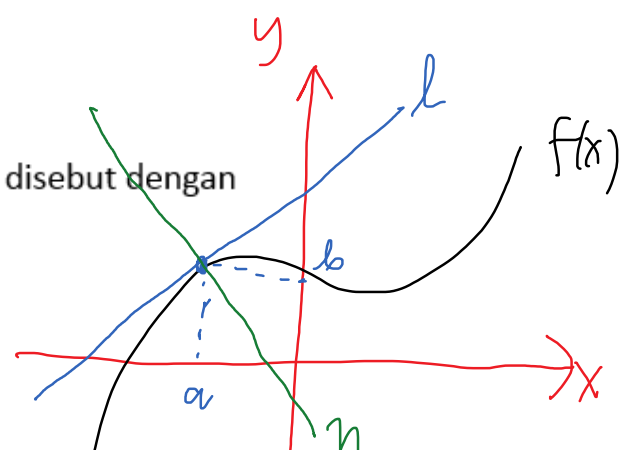
$(1 + x \cos(xy)) \frac{dy}{dx} = -y \cos(xy) \Rightarrow \frac{dy}{dx} = \frac{-y \cos(xy)}{1 + x \cos(xy)}$

- Persamaan garis singgung fungsi  $y = f(x)$  di titik  $(x_0, y_0)$  dengan kemiringan  $m$  adalah

$y - y_0 = m(x - x_0)$

- Garis yang tegak lurus dengan garis singgung disebut dengan garis normal
- Persamaan garis normal di titik  $(x_0, y_0)$  adalah

$y - y_0 = -\frac{1}{m}(x - x_0)$



l garis singgung

$y - b = f'(a)(x - a)$

n garis normal

Diketahui kurva yang dinyatakan secara implisit

$y + \sin(xy) = 1$

Tentukan persamaan garis singgung dan garis normal di  $(\pi, 1)$

$\frac{dy}{dx} = ?$

$x = \pi$

$$\left. \frac{dy}{dx} \right|_{x=\pi} = ?$$

$$[x=\pi]$$

m

m garis normal

$$1 dy + \cos(xy) \cdot y dx + \cos(xy) \cdot x dy = 0 \quad y-b = -\frac{1}{f'(a)}(x-a)$$

$$\frac{dy}{dx} + y \cos(xy) + x \cos(xy) \frac{dy}{dx} = 0$$

$$\left( \frac{\pi}{1}, 1 \right) \rightarrow x=\pi \rightarrow \frac{dy}{dx} + 1 \cdot \cos \pi + \pi \cos \pi \frac{dy}{dx} = 0$$

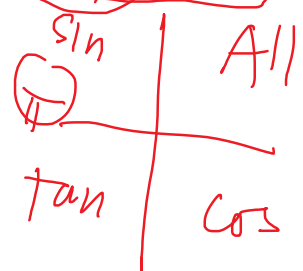
$$\frac{dy}{dx} - 1 + \pi(-1) \frac{dy}{dx} = 0$$

$$(1 - \pi) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{1-\pi} = m \text{ (gradien)}$$

Pers. grs. singgung  $\rightarrow y-1 = \frac{1}{1-\pi}(x-\pi)$

" " normal  $\rightarrow y-1 = -(1-\pi)(x-\pi)$



1. Diketahui kurva yang dinyatakan secara implisit :

$$x^2 + xy + y^2 - 3y = 10$$

Tentukan

- a. Turunan pertama di  $x=2$
- b. Persamaan garis singgung dan normal di  $x=2$

$$x=2 \rightarrow y = ? \Rightarrow 2^2 + 2y + y^2 - 3y = 10$$

$$y^2 - y - 6 = 0$$

$$(y+2)(y-3) = 0$$

$$y = -2, y = 3$$

Berarti ada dua titik yaitu  $(2, -2)$  dan  $(2, 3)$

$$2x dx + y dx + x dy + 2y dy - 3 dy = 0$$

$$2x + y + x \frac{dy}{dx} + 2y \frac{dy}{dx} - 3 \frac{dy}{dx} = 0$$

$$(2, -2) \rightarrow 4 - 2 + 2 \frac{dy}{dx} - 4 \frac{dy}{dx} - 3 \frac{dy}{dx} = 0$$

$$-5 \frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = \frac{2}{5}$$

Pers. grs. singgung  
Pers. grs normal

$$y + 2 = \frac{2}{5} (x - 2)$$
$$y + 2 = -\frac{5}{2} (x - 2)$$

$$(2, 3) \rightarrow$$