Limit Fungsi

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Bila x mendekati a dari arah kiri ($x \to a^-$) dan nilai fungsi $f(x)$ mendekati L ($f(x) \to L$), maka ditulis kan $\lim_{x \to a^-} f(x) = L$ → limit kiri

Bila x mendekati a dari kanan ($x \to a^+$) dan nilai fungsi $f(x)$ mendekati k ($f(x) \to k$), maka

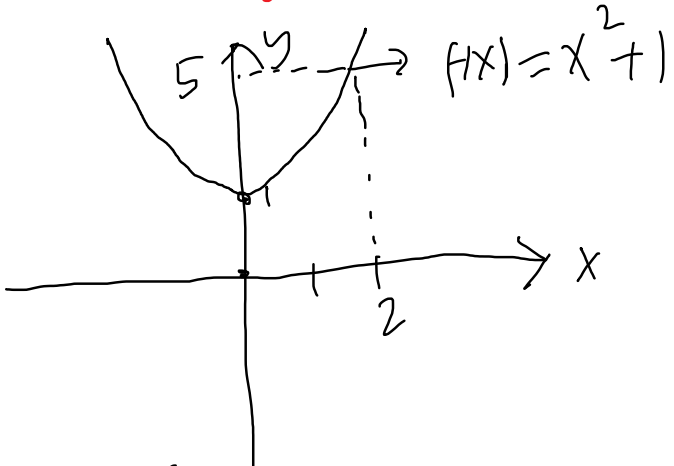
$\lim_{x \to a^+} f(x) = k$ → limit kanan

$\lim_{x \to a} f(x)$ { ada, $L = k$; tidak ada, $L \neq k$

contoh

① $\lim_{x \to 2} (x^2 + 1) = 5$

$f(x)$ → grafiknya parabola



② $\lim_{x \to 1} f(x)$, dg $f(x) = \begin{cases} 2x - 1, & x \geq 1 \\ x, & x < 1 \end{cases}$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1$ ∩

$x \to 1^-$ $\quad = \quad x \to 1^-$ $\quad$ dst =

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x-1) = 1$$

Jadi $\lim_{x\to 1} f(x) = 1$

Contoh

Diketahui

$$f(x) = \begin{cases} 2x - a & ; x < -3 \\ ax + 2b & ; -3 \le x \le 3 \\ b - 5x & ; x > 3 \end{cases}$$

Tentukan nilai a dan b agar

$\lim_{x \to 3} f(x)$

dan

$\lim_{x \to 3} f(x)$

ada.

$\lim_{x\to -3} f(x)$ Ada $\Leftrightarrow \lim_{x\to -3^-} f(x) = \lim_{x\to -3^+} f(x)$

$$\lim_{x\to -3^-} (2x-a) = \lim_{x\to -3^+} (ax+2b)$$

$$-6-a = -3a+2b \quad (1)$$

$\lim_{x\to 3} f(x)$ Ada $\Leftrightarrow \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$$\lim_{x\to 3^-} (ax+2b) = \lim_{x\to 3^+} (b-5x)$$

$$3a+2b = b-15 \quad (2)$$

Dari (1) dan (2)

$-6 = -2a + 2b \to -3 = -a+b$

$-15 = 3a + b \to -15 = 3a+b$

$12 = -4a$

$a = -3$

$-15 = -9 + b \to b = -6$

Kekontinuan fungsi

$f(x)$ kontinu di $x=a$ bila $\lim_{x\to a} f(x) = f(a)$

① $f(x) = \dfrac{x^2-3x+2}{x^2-1}$. Apakah $f(x)$ kontinu di $x=1$?

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2-3x+2}{x^2-1} = \lim_{x\to 1} \frac{(x-2)(x-1)}{(x+1)(x-1)}$$

$$\frac{5\times 2}{4\times 2} = \frac{5}{4}$$

$$= \lim_{x\to 1} \frac{x-2}{x+1} = -\frac{1}{2}$$

$$\frac{5\times 0}{4\times 0} = \quad f(1) = ? \rightarrow f(1) \text{ tidak terdefinisi.}$$

Jadi $f(x)$ diskontinu di $x=1$

Selidiki apakah fungsi

$$f(x) = \begin{cases} 2x-1, & x \geq -1 \\ \dfrac{x^2-2x-3}{x+1}, & x < -1 \end{cases}$$

kontinu di titik x = -1

a) $f(-1) =$

b) $\lim_{x\to -1} f(x) =$?

$f(-1) = 2(-1)-1 = -3$

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$$

$$\lim_{x\to -1^-} \frac{x^2-2x-3}{x+1} = \lim_{x\to -1^+} (2x-1)$$

$$\lim_{x\to -1^-} \frac{(x-3)(x+1)}{x+1} = -3$$

$-4 \neq -3$

→ diskontinu di $x=-1$

Agar $f(x)$ kontinu di $x=-1$

Tentukan nilai k agar fungsi

$$f(x) = \begin{cases} \dfrac{x^2 + 2kx + 1}{x + 1}, & x < -1 \\ x^2 - 1, & x \geq -1 \end{cases}$$

kontinu di x = -1.

Agar $f(x)$ kontinu di $x = -1$ maka syaratnya

$$f(-1) = \lim_{x \to -1} f(x)$$

Sebab $f(-1) = \lim_{x \to -1^+} f(x)$, maka syaratnya dapat dituliskan,

$$f(-1) = \lim_{x \to -1^-} f(x)$$

$$(-1)^2 - 1 = \lim_{x \to -1} - \frac{x^2 + 2kx + 1}{x + 1}$$

$$0 = \lim_{x \to -1} - \frac{(x + 2k - 1)(x + 1)}{x + 1}$$

$$0 = -1 + 2k - 1$$

$$2k = 2$$

$$k = 1$$

$x + 2k - 1$

$x + 1 \;|\; x^2 + 2kx + 1$

$x^2 + x$

$2kx + 1 - x$

$(2k - 1)x + 1$

$(2k - 1)x + 2k - 1$

$1 - (2k - 1) = -2k + 2$

$0 \quad k = 1$

(20) (4) $= 5 + \text{sisa } 0$ $\quad k = 1$