Definisi Turunan

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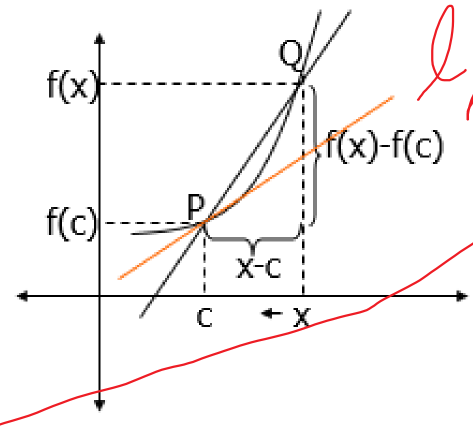
Gradien garis PQ dapat dinyatakan dengan,

$$m_{PQ} = \frac{f(x)-f(c)}{x-c}$$

Kemiringan garis



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Gradien garis singgung PQ dinyatakan dengan,

$$\boxed{m = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}}$$

$y = 4x - 5 \to m = 4$

$x - 2y + 5 = 0 \to m = \frac{1}{2}$

$-2y = -x - 5$

$y = \frac{1}{2}x + 5/2$

→ definisi dari turunan

Misal diberikan $y = f(x)$. Maka turunan dari $y = f(x)$ di $x = c$ didef

$$f'(c) = \boxed{\lim_{x\to c}\frac{f(x)-f(c)}{x-c}} = \left.\frac{dy}{dx}\right|_{x=c} = \left.\frac{df(x)}{dx}\right|_{x=c}$$

$x - c = h$

$x = c + h$

$$f'(c) = \lim_{x-c\to 0}\frac{f(x)-f(c)}{x-c} = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

Bila lim tidak ada maka $f(x)$ tidak diferensiable di $x = c$

Bila lim ada maka $f(x)$ dsb diferensiabel di $x = c$

contoh: $f(x) = x^2 \to \cancel{f'(2) = 2\cdot 2 = 4}$

$\cancel{f'(x) = 2x}$

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{(2+h)^2-4}{h} = \lim_{h\to 0}\frac{4+4h+h^2-4}{h}$$

$$= \lim_{h\to 0}\frac{4h+h^2}{h} = \lim_{h\to 0}(4+h) = 4$$

Turunan kiri dari fungsi f di titik c, didefinisikan sebagai :

$$f'_-(c) = \lim_{x \to c^-} \frac{f(x) - f(c)}{x - c}$$

Turunan kanan dari fungsi f di titik c, didefinisikan sebagai :

$$f'_+(c) = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$$

Fungsi f dikatakan mempunyai turunan(diferensiabel) di c atau $f'(c)$ ada, jika

$$f'_-(c) = f'_+(c) \text{ dan } f'(c) = f'_-(c) = f'_+(c)$$

contoh: $f(x) = \begin{cases} 2x - 1 , & x \le 1 \\ 3x , & x > 1 \end{cases} \rightarrow$ tidak dif. di $x = 1$

Apakah $f(x)$ diferensiabel ~~di~~ $x = 1$ ..?

$$f'_-(1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{(2x - 1) - 1}{x - 1} = \lim_{x \to 1^-} \frac{2x - 2}{x - 1}$$

$$= \lim_{x \to 1^-} \frac{2\cancel{(x - 1)}}{\cancel{x - 1}} = \boxed{2}$$

$$f'_+(1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{3x - 1}{x - 1} = \lim_{x \to 1^+} \frac{3x - 3 + 2}{x - 1}$$

$$= \lim_{x \to 1^+} \left(\frac{3x - 3}{x - 1} + \frac{2}{x - 1} \right) = 3 + \infty = \boxed{\infty}$$

$f(x) = \begin{cases} 2x - 2 , & x \ge 2 \\ x , & x < 2 \end{cases} \rightarrow$ apakah $f(x)$ dif di $x = 2$?

$$f'_-(2) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{\cancel{x - 2}}{\cancel{x - 2}} = 1$$

tidak dif

$$f'_+(2) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{(2x - 2) - 2}{x - 2} = \lim_{x \to 2^+} \frac{2x - 4}{x - 2} = 2$$

Tentukan konstanta a dan b agar fungsi f(x) berikut diferensiabel di x=1 ;

$$f(x) = \begin{cases} x^2 + b , & x < 1 \\ ax , & x \ge 1 \end{cases}$$

Agar f(x) terdiferensialkan di $x = 1$, haruslah

a. f kontinu di $x = 1$ (syarat perlu)

b. Turunan kiri = turunan kanan di $x = 1$ (syarat cukup)

a) f(x) kontinu di $x = 1$ bila $\boxed{f(1) = \lim_{x \to 1} f(x)}$

$$f(1) = \lim_{x \to 1^-} f(x)$$

$$a = \lim_{x \to 1^-} (x^2 + b)$$

$$\boxed{a = 1 + b}$$

b) $f'_-(1) = f'_+(1)$

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\lim_{x \to 1^-} \frac{(x^2 + b) - a}{x - 1} = \lim_{x \to 1^+} \frac{ax - a}{x - 1}$$

$$\lim \frac{x^2 + \boxed{b - a}}{} = \lim_{+} \frac{a\cancel{(x - 1)}}{}$$

$a = 1 + b$

$$\boxed{b - a = -1}$$

$$\boxed{b = 1}$$

$$\lim_{x \to 1^-} \frac{x^2 + \boxed{b - a}}{x - 1} = \lim_{x \to 1^+} \frac{a\cancel{(x-1)}}{\cancel{x-1}}$$

$$\boxed{b = 1}$$

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = a$$

$$\lim_{x \to 1^-} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = a$$

$$\boxed{2 = a}$$

Tentukan nilai *a* dan *b* agar fungsi berikut diferensiabel di titik yang diberikan.

1. $f(x) = \begin{cases} a\sqrt{x+3} & ; 0 \le x < 1 \\ x^2 - bx & ; x \ge 1 \end{cases}$, $x = 1$

2. $f(x) = \begin{cases} ax - b & ; x < 2 \\ 2x^2 - 1 & ; x \ge 2 \end{cases}$, $x = 2$

3. $f(x) = \begin{cases} x^2 - 1 & ; x < 3 \\ 2ax + b & ; x \ge 3 \end{cases}$, $x = 3$